Math Links

PROFICIENCY CHALLENGES
GRADE 8
SETS 9-12

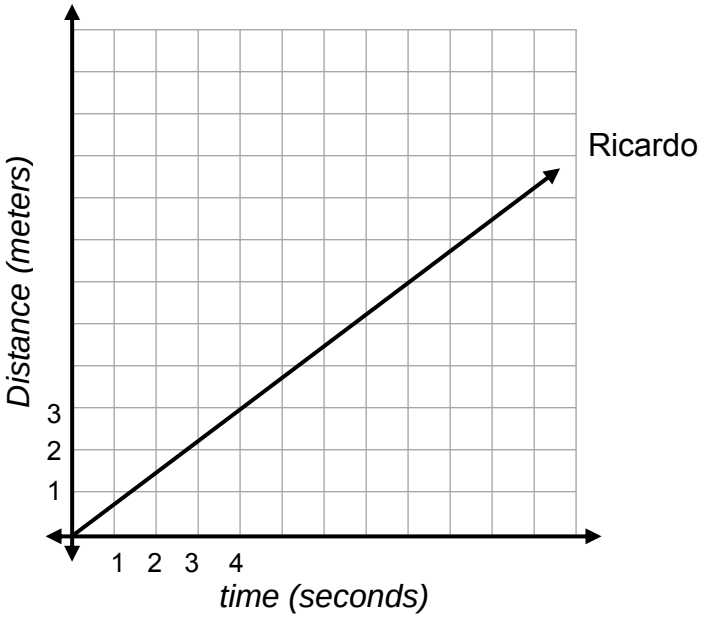
Proficiency Challenges are sets of interesting, mixed-topic problems. It may take a lot of time to complete each set, so consider doing only one or two parts at a time.

Name_____ Period_____ Date _____

PROFICIENCY CHALLENGE 9

Complete each problem on your own paper. Show all work. Use graph paper if needed.

1. Ricardo had a race with his younger sister, Paola. Ricardo's distance versus time is shown on the graph to the right.



   a. Find a linear equation ($y = mx + b$) that represents Ricardo's graph.

   b. Paola's distance x seconds into the race can be expressed as: $y = \frac{1}{2}x + 2$.

      Draw a graph of this line on the grid.

   c. Ricardo gave Paola a head start. How much distance did Ricardo give Paola as a head start? Explain your reasoning.

   d. Under these conditions, if Ricardo wanted the race to end in a tie, how long (in meters) should the race be? Explain your reasoning.

2. A representative sample of 20 students from a middle school is surveyed. Each student was asked: "What is your favorite sport?" The table shows the responses.

Sport	Baseball	Swimming	Football	Volleyball	None
Number of Students	8	3	6	1	2

   Identify whether each statement is valid based on survey results.

   a. The number of students who chose swimming was twice the number of those who chose football.

   b. The number of students who chose no sport was twice the number who chose volleyball.

   c. 80% of the students prefer baseball.

   d. In a group of 100 students with similar preferences, one would expect 15 students to prefer swimming.

Name_____ Period_____ Date _____

PROFICIENCY CHALLENGE 9 (Continued)

3. Consider the equation $4(x + 2) – 3(2x + 5) = x + 8 + y$

   Choose all expressions that are equivalent to y?

   $-3x + 15$ $-3x – 15$ $-3(x + 5)$ $3(-x + 5)$ $-3(x + 3)$ $3(-x – 5)$

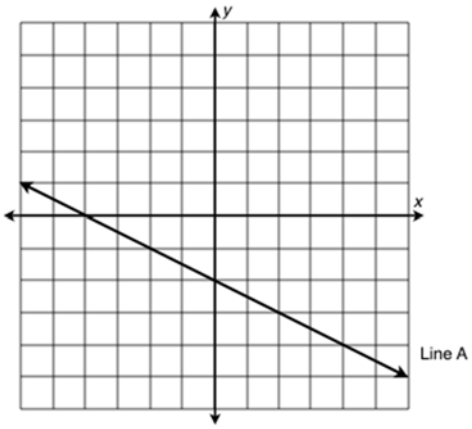
4. Kuang and Han wrote down two **different** functions that have the same rate of change. Kuang's function is represented in the table at the right.

   On a coordinate plane, graph a function that could be Han's function.

Kuang's Table

x	y
0	-6
3	3
2	0

5. Refer to the graph at the right.



   a. Find the linear equation of Line A.

   b. Graph a second line called "Line B" so that the solution to the system of equations for Line A and Line B is (2, -3).

   c. Find the equation for your Line B.

   d. Show that (2, -3) satisfies both the linear equations for Line A and Line B.

Name _____ Period_____ Date _____

PROFICIENCY CHALLENGE 10

Complete each problem on your own paper. Show all work. Use graph paper if needed.

1. Joe solved this linear system.

$$\begin{cases} 6x + 3y = 6 \\ y = -2x + 4 \end{cases}$$

Here is his work.

$$\begin{aligned} 6x + 3(-2x + 4) &= 6 \\ 6x - 6x + 12 &= 6 \\ 12 &= 6 \end{aligned}$$

Choose all statements that are true about this system.

A. x must equal 6.
B. y must equal 6.
C. There is no solution to this system.
D. There are infinitely many solutions to this system.
E. There is an error in Joe's work.

2. A bag contains four different ping pong balls with the numbers 0, 2, 4, and 6 on them. One ball is pulled from the bag at random, the number is noted, and the ball is replaced. A second ball is pulled and its number is noted. What is the probability that the product is of the two numbers is less than 10?

3. Choose all expressions that can be placed in the box to make this a true equation for all values of x.

$$-(\square) = x - (2x + 3)$$

A. $x + 3$ B. $-x - 3$ C. $-x + 3$ D. $-(x - 3)$

4. Graph a system of two linear equations that has a single solution of (-2, 4). Write the linear equations. Show that their solution is (-2, 4) algebraically.

Name_____ Period_____ Date _____

PROFICIENCY CHALLENGE 11

Complete each problem on your own paper. Show all work. Use graph paper if needed.

1. The following numbers represent gallons of water.

   8×10^{10} 7×10^{5} 11 2×10^{-2}

   Which number best represents the volume of each example below? Explain your reasoning for each answer.

   a. The amount of water in a human body.
   b. The amount of water that the average cat drinks in a day.
   c. The amount of water in a lake.
   d. The amount of water in an Olympic swimming pool.

2. The mass of our sun is about 2×10^{30} kg. A star with a mass of 6×10^{34} kg is how many times more massive than the sun?

3. Jorge was working on some astronomy problems and multiplied two large numbers together. His calculator said the product was: 4.2 E 19

   Express this number in scientific notation. Assuming that both numbers were bigger than one billion, what could have been the two numbers that Jorge multiplied together? Explain.

4. Valerie and Tatiana agree that the value of $\sqrt{23}$ is in between 4 and 5. Valerie thinks that $\sqrt{23}$ is closer to 4. Tatiana thinks that $\sqrt{23}$ is closer to 5.

   a. Which student do you think is more accurate?
   b. What would you say to the other student to help her understand her error?

5. Write TWO expressions equal to 64 that are in the form $2^m \cdot 2^n$, where m and n are integers.

6. Write x^{-4} in THREE different but equivalent ways that include exponents.

7. Write a numerical expression with a negative base and a negative exponent that has a positive value. Show your work and explain your reasoning.

8. Write each expression in exponent form (b^n). For problems b and c, $x \neq 0$.

   a. $\dfrac{(6^3)^4}{(6^5)(6^1)}$ b. $\dfrac{x^2 \cdot x^4}{x^9}$ c. $\dfrac{\left(x^4\right)^3}{x^{13}}$

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PROFICIENCY CHALLENGE 12

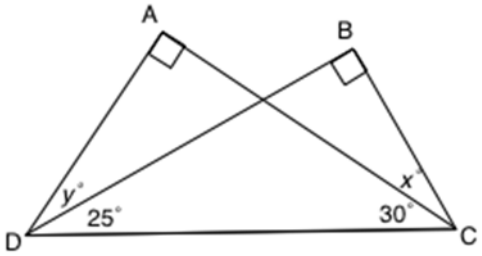
Complete each problem on your own paper. Show all work. Use graph paper if needed.

1. Right triangles ΔACD and ΔBCD overlap as shown in the diagram on the right.

   The measure of $\angle BDC$ is 25°.

   The measure of angle $\angle ACD$ is 30°.

   Find the values of x and y.



2. A right triangle has hypotenuse of 15 units long and each leg is a whole number of units long. Sketch the triangle and label the lengths of each leg. Explain your reasoning.

3. A square, with side length x has an area of 324 square centimeters. What is the side length of the square in centimeters?

4. A box in the shape of a cube has edges that are each 8 inches long. Will a 12-inch ruler fit inside the box? Explain your reasoning.

5. Carmen asked 20 students in her class whether they had a bike, a skateboard, or both.

   Complete the table with the given information.

   - 60% of the students had a bike.
   - 20% of the students had a skateboard.
   - 10% of the students had both a bike and a skateboard.

	Bike	No Bike	Total
Skateboard			
No Skateboard			
Total			

6. Ingmar had to complete the following tasks. Explain to him how he should approach each task, including appropriate tools he might use. Do not do each task.

   a. Find the length of the line segment at the top of the page after "Name."
   b. Find the length of the line segment with endpoints (0, 0) and (3, 2).
   c. Multiply 40 • 60.
   d. Multiply 863 • 9,142.
   e. List the dimensions of all rectangles with whole number side lengths that have an area equal to 36 square units.